RATIONAL ELASTIC ANALYSIS AND DESIGN OF COLD-FORMED LIGHT GAUGE STEEL MEMBERS

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By

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CERTIFICATE

This is to certify that the thesis entitled 'RATIONAL ELASTIC ANALYSIS AND DESIGN OF COLD-FORMED LIGHT GAUGE STEEL MEMBERS' is a record of work carried out under my supervision by Raghuram, E. and has not been submitted elsewhere for a degree.

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ABSTRACT

Modifications in the present design procedure for cold-formed light gauge steel members are suggested towards achieving a more rational design. These modifications achieve a considerable amount of economy in the design of cold-formed members. This aspect is brought out with illustrations and as functions of the various parameters. A rational and a practical design algorithm has been developed for the design of cold-formed continuous beams. Design charts are also given.

ACKNOWLEDGEMENT

I do not propose to thank my supervisor, Dr. Sri Ranga Sai Adidam, for the help I have received from him, academic and otherwise, for to thank him adequately would be well nigh impossible.

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RAGHURAM, E .

LIST OF SYMBOLS

a, c	non-dimensional parameters fixing the section dimensions in the unreduced state
a_e, c_e	non-dimensional parameters fixing the section dimensions in the reduced state
b_e	reduced width of an element
f	stress for which b_e is calculated
f_a	allowable stress on flanges
f_1^c	compressive stress on the flange under the action of M_1^+
f_2^c	compressive stress on the flange under the action of M^-
f_3^c	compressive stress on the flange under the action of M_2^+
h, h_e	depth of the section in the unreduced and reduced state respectively
k	constant depending on the support conditions of the element
l	span of the beam
l_1, l_2	spans of the continuous beam
q	intensity of loading
r	radius of gyration of the unreduced section
t	thickness of the element
w	width of the element
$(w/t)_{lim}$	limiting value of (w/t) for a stress 'f'
y_c, y_t	non-dimensional parameters fixing the location of the neutral axis in flexural members
y_{max}	maximum deflection in span 1

A_B	area of the bottom flange
A_S	area of stiffened elements
A_T	area of top flange
A_U	area of unstiffened elements
C_c	constant depending on the material properties E, F_y
E	Young's modulus of the material of the element
F_{all}	allowable average stress for the section
F_y	yield stress
I	moment of inertia of the section
I_1, I_2, I_3	moments of inertia of the beam in the different segments
KL	effective length of the column member
M, M^-	negative bending moment over the intermediate support
M_1^+, M_2^+	maximum value of positive bending moments (subscript refers to the span)
$M(x)$	bending moment as a function of coordinate x along the length of the beam
M_{EW}, M_{SR}	moment capacities calculated by effective width and stress reduction approaches respectively
P_{EW}, P_{SR}	axial load capacities calculated by effective width and stress reduction approaches respectively
Q	form factor
Q_a , Q_s	area and stress form factors respectively
R_1	ratio of moments of inertia I_1 and I_2
R_2	ratio of moments of inertia I_2 and I_3
R_A, R_{B1}, R_{B2}, R_C	reactions at the supports of the continous beam

Z_c	section modulus with respect to compression
α, ε	nondimensional parameters fixing the length of different segments in the continuous beam
α_{y1}	ratio fixing the position of maximum deflection
β	ratio of spans l_1 and l_2
σ_{av}	average stress on the whole element
σ_{cr}	local buckling stress
σ_{emax}	maximum edge stress
ν	Poisson's ratio

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CHAPTER 1

INTRODUCTION

1.1 General:

In the western countries, where labour is hard to come by, steel was considered to be the most convenient material for structural purposes. Steel was hot rolled into standard sections for use in structures. Later thin sheets of steel were cold-formed to get light gauge sections.

To be precise use of cold-formed sections is not really a new development. Corrugated sheets have been used for many decades. But no systematic study was undertaken to understand the behaviour of cold-formed members before 1930's. In the United States of America large scale use of light gauge sections started only after the second world war. Countries in Europe have taken to these structures only presently. Use of light gauge sections for structural purposes is at its embryonic stage in India, although the code of practice, IS 801-1958 was published as early as 1958.

Cold-formed light-gauge members are cold formed from flat steel with thicknesses between 0.8-4.0 mm. Forming is done either in press brakes or by cold rolling. The latter is used when mass production is undertaken for a well-established and standardised section. Cold formed members

are preferred to hot rolled sections in the following situations: (i) where moderate loads and spans render the thick hot rolled sections uneconomical (ii) where members are required of cross sections which cannot economically be produced by hot rolling (iii) where useful surfaces are desired of a structural member.

In general cold-formed sections have outlines similar to that of hot rolled sections. However, ease of fabrication on press brakes lead to a wide variety of shapes which are better than the conventional shapes in some respects. The conventional hot rolled section 'I', can be conveniently made of cold formed members by welding two channel sections back to back.

The hot rolled sections and cold formed sections supplement each other in the making of a building; the joists, floors, roofs can be made out of cold formed sections and columns and other framing elements out of hot rolled sections.

1.2 State of Art:

Presently the United States of America (USA), United Kingdom (UK), some European countries, and India have their own specifications and codes of practice for cold-formed members. The Indian Standard IS 801-1975 (1)* is based almost entirely on the work done in the United States and

* Number inside bracket indicates reference

also the specifications on this topic published by American Iron and Steel Institute (AISI).

As this work is going to be on individual structural sections rather than on panels and decks, it is pertinent to discuss the state of the art regarding individual sections. Designing of a cold formed section for tensile forces does not pose any problem different from those encountered in the designing of hot rolled sections. Whereas, the behaviour of cold formed members is radically different from the behaviour of hot rolled members under compression or bending. Too many additional factors have to be taken into account in the case of cold formed members to get a safe design as the cross section is thin walled.

Any section of cold formed members can be split into the following categories of elements: (i) stiffened elements (ii) unstiffened elements. Stiffened elements are those wherein both the edges parallel to the direction of stress are supported and stiffened either by a web or by a stiffening lip. The unstiffened elements are stiffened only on one edge parallel to the direction of stress.

The stiffened elements have additional load carrying capacity even after buckling has occured. This is called the post buckling strength. This strength is utilized by making use of the effective width approach initially proposed by

Theodore v.Karman. In the post buckling stage the width of the element is taken less than the actual width and this reduced width is subjected to the maximum stress the material can carry before yielding. In general the unstiffened elements donot exhibit as much postbuckling strength as stiffened elements and also the out-of-plane deformations are more in the post buckling stage in the case of unstiffened elements. Hence the local buckling stress (σ_{cr}) is taken as the limiting stress the element can carry. This stress value divided by the factor of safety gives the allowable stress on the element.

Recent research (2) shows that the post buckling strength of unstiffened elements is not insignificant and this can be taken care by the effective width approach. An effective width equation has been developed based on a number of tests on unstiffened elements (2). Though the actual effective width equation for unstiffened and stiffened elements differ, the basic approach will be the same making the concept of design easier.

A peculiar situation arises in cold formed flexural members. The moment of inertia 'I' differs from section to section in a flexural member subjected to a non-uniform bending moment along its length. This is because the effective dimensions of the section are fixed by the bending

moment; to be more precise by the stress caused by the bending moment in the elements. In the present design practice the moment of inertia 'I' of the beam is taken to be constant along the length and this value of I is fixed with respect to the maximum bending moment along the length. This approach is conservative as long as the bending moment does not change sign along the length.

If the bending moment changes sign along the length, the position of the neutral axis and the moment of inertia should be taken to be constant only over the segments of the length where the moment is of the same sign. In each of these segments the moment of inertia should be fixed for the maximum bending moment occuring in that segment. If this is not done these sections based on the maximum sagging (or hogging) bending moment can be unsafe in the hogging (or sagging) moment region. The values of shear forces and bending moments will remain the same over the entire structure irrespective of the values of the moments of inertia in various segments if the structure is statically determinate.

If the structure is statically indeterminate the analysis itself will suffer. Gjelsvik and Bleustein (3) have deviced an alogrithm by which this problem can be tackled and they have successfully applied this to analysis of continuous beam, continuous over one support with equal spans,

simply supported at the extreme supports.

1.3 Scope of the Thesis:

It was pointed out that designing unstiffened elements using the concept of effective width will lead to a unified approach in the design of both stiffened and unstiffened elements. For various reasons, enumerated else where, it was felt that effective width approach to unstiffened elements would lead to a better utilisation of a section in terms of strength. So the capacities of a section as obtained from both the approaches are compared. These comparisons are done for both beam and column members.

The algorithm developed by Bleustein and Gjelsvick (3) ends up in finding a design length for a given section. As the loads and spans are the design parameters this algorithm will not be useful in day to day design problems. So a modification thereof suited for design purposes has been suggested. The algorithm has been worked out for continuous beams of unequal spans loaded uniformly. Charts useful for design purposes are also given.

CHAPTER 2

UNSTIFFENED COMPRESSION ELEMENTS

2.1 General:

Cold-formed members are made up of unstiffened and stiffened elements. Using the present design provisions adopting a reduction of stress over the whole area of the unstiffened element, one almost always arrives at an oversafe, conservative design. This conservatism is very much in certain cases and is totally unjustifiable.

In this chapter the effective width approach and the stress reduction approaches are applied to find the capacities of section under compression and bending and a comparison of the same is effected.

2.2 Buckling Characteristics:

The local buckling stress of an ideally flat compression element is given by the following formula:

$$\sigma_{cr} = \frac{k\,\pi^2 E}{12(1-\nu^2)\left(\frac{w}{t}\right)^2} \qquad (2.1) \quad (\text{Ref.}(4))$$

where,

σ_{cr} - local buckling stress,

E - Young's modulus of the material of the element,

ν - Poisson's ratio of the material of the element, taking as 0.3,

$(\frac{w}{t})$ - Width to thickness ratio of the element

k - Constant which depends on the end rotational restraint offered to the element at the stiffened edge.

The element can be idealised as shown in Fig. 2.1. The idealised element undergoes a bifurcation type of buckling (i.e.) the element remains flat till the stress on the element reaches σ_{cr}. Beyond the local buckling stress, the element undergoes out-of-plane deformations and the stress distribution is non-uniform. In practice, bifurcation type of buckling is absent due to initial imperfections and hence out-of-plane deformations begin at a stress much lower than the local buckling stress.

2.3 Methods for Taking the Post Buckling Strength into account:

At this stage two methods can be adopted to find the load taken by the element. Either an average stress, σ_{av}, can be taken to be acting on the whole element or the maximum edge stress (σ_{emax}) can be taken to be acting on a reduced width (b_e). The former method is being adopted in the codes of practice.

The stress reduction approach has a few inherent deficiencies. In compression members having an unstiffened element with a large width to thickness (w/t) ratio, the stress level of the whole section gets reduced and consequently

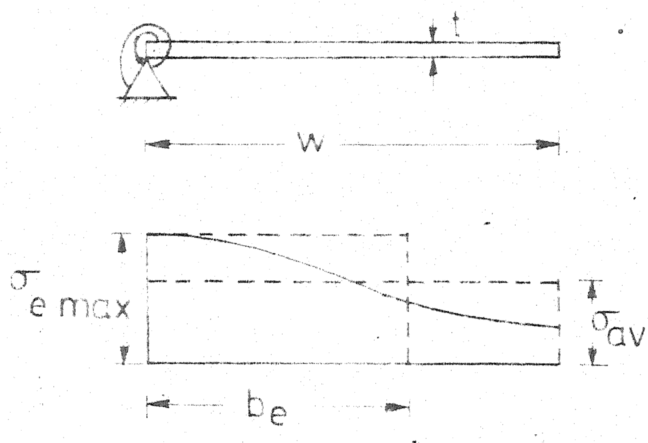

FIG.2.1 IDEALISED UNSTIFFENED ELEMENT

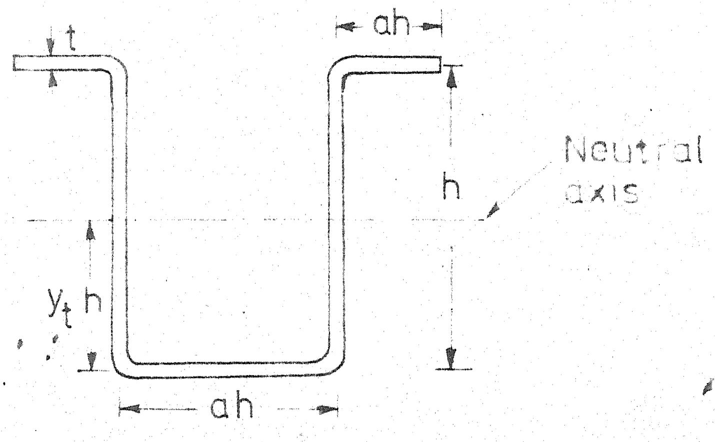

FIG.2.2 SECTION FOR FLEXURE

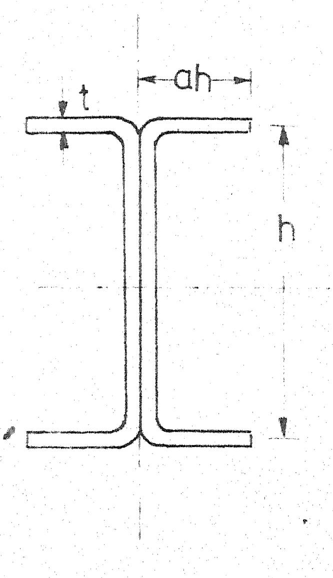

FIG.2.3 SECTION FOR AXIAL LOAD

the capacity is reduced. This leads to an unjustifiably conservative design. In the effective width approach, though the form factor Q (as no reduction of stress is employed Q equals area form factor Q_a) gets reduced, the stress levels in stiffened elements remain at their highest and ultimately one can get a higher capacity from the same section. The area form factor Q_a, the stress form factor Q_s, and the form factor Q are as defined in Ref.(1).

2.4 Effective Width Equation for Unstiffened Elements:

The basic equation for effective width is of the following form (2):

$$\frac{b_e}{w} = 1.19 \sqrt{\frac{\sigma_{cr}}{\sigma_{emax}}} \left(1.0-0.298 \sqrt{\frac{\sigma_{cr}}{\sigma_{emax}}}\right) \qquad (2.2)$$

where,

w - unreduced width of the element

b_e - reduced width of the element

σ_{emax} - maximum edge stress

The value of the constant k in equation(2.1) is taken to be 0.5 to be consistent with the value used in the stress reduction approach. This is a conservative value since experiments indicate a consistently high value (2). For small (w/t) ratios this constant k does not exceed 0.5 by a large amount.

Substituting for σ_{cr} in equation(2.2) from equation (2.1) the following formula is obtained:

$$\frac{b_e}{t} = \frac{1146}{\sqrt{f}} \left(1.0 - \frac{287.8}{(w/t)\sqrt{f}} \right) \qquad (2.3)$$

where, 'f' is synonymous with σ_{emax} and 't' is the thickness of the element . The maximum value of 'f' is 0.6 F_y where, F_y is the yield strength of the material of the element in tension. As pointe out in article C.2(c) of Ref.(5), if '1.67f' is substituted for 'f' in equation (2.3), the equation for load determination is obtained.

$$\frac{b_e}{t} = \frac{886.8}{\sqrt{f}} \left(1.0 - \frac{222.7}{(w/t)\sqrt{f}} \right) \qquad (2.4)$$

The equation (2.4) is the effective width equation for load determination. The corresponding $(w/t)_{limit}$ below which the entire element is effective for a particular stress 'f' is obtained by putting $b_e/t = w/t$ in equation (2.3).

$$(w/t)_{lim} = \frac{532.9}{\sqrt{f}} \qquad (2.5)$$

For load determination

$$(w/t)_{limit} = \frac{412}{\sqrt{f}} \qquad (2.6)$$

If 0.6 F_y is substituted for 'f' in equations (2.5) and (2.6) one gets the $(w/t)_{lim}$ below which the elements will be fully effective upto design stress. For F_y=2320 kg/cm^2 equation (2.6) yields $(w/t)_{lim}$ = 10.8.

2.5 Comparison for Flexural Members:

Frequently, finding out the moment capacity of a section is a trial and error problem in cold-formed members. Moreover, the equations used for finding the reduced stress and effective widths are semiempirical making algebraical manipulations difficult. Therefore, the comparison of capacities obtained from the two approaches is effected by numerical examples (i.e.) sections were fixed and capacities found out and compared. The parameters of the section which were most likely to affect the comparison were changed to obtain a clearer picture.

In general a hat section or a closed box section is an ideal choice for flexural members. Members of these sections have a high resistance to lateral buckling. Open hat sections are better than closed box sections because of ease of fabrication. Hence open hat sections are considered in the subsequent work with their closed portion on tension side.

Henceforth the effective width and the stress reduction approaches shall be abbreviated to EW and SR respectively.

The notation for section dimensions are shown in Fig. (2.2). The various dimensions of the section are

expressed as fractions of the depth 'h'

a - ratio of width of tension flange to the depth

c - ratio of width of unstiffened flange on the compression side to the depth

y_t - ratio of the distance to the neutral axis from the tension flange to the depth.

A model calculation is shown below.

The following section dimensions are chosen;

$h = 30$ cms ; $t = 0.25$ cms ; $a = 0.3$; $c = 0.125$

Total area of the cross section , $A = 19.125\ cm^2$. Then,

$$\frac{ah}{t} = 36 \ ; \quad \frac{ch}{t} = 15.$$

(a) SR Approach:

Allowable stress for top flange (T) is given by

$$f_a = F_y \left(0.767 - \frac{3.15}{10^4} (w/t)\sqrt{F_y}\right) \quad \text{(Section 6.2 of Ref.(1))}$$

$$= 1251\ kg/cm^2$$

(Steel with $F_y = 2320\ kg/cm^2$ is used)

$$y_t = \frac{2c + 1}{a + 2c + 2} = 0.490$$

Stress in bottom flange (B) $= f_a \times \dfrac{y_t}{(1-y_t)} = 1202\ kg/cm^2$

Moment capacity by SR approach is given by

$$M_{SR} = f_a\, h^2 t \left(2c(1-y_t) + \frac{2}{3}(1-y_t)^2 + \frac{a\, y_t^2}{(1-y_t)} + \frac{2\, y_t^3}{3(1-y_t)}\right)$$

$$= 1.68\ tm$$

(b) EW Approach:

As y_t is less than 0.5 the T flange yields first.

$f_a = 1450 \text{ kg/cm}^2$

$$\frac{c_e h}{t} = \frac{886.8}{\sqrt{1450}} \left(1.0 - \frac{222.7}{15\sqrt{1450}}\right) = 14.20; \; c_e = 0.118$$

where, $c_e h$ - effective width of T flange at stress 'f_a'

$y_t = 0.487$

Moment capacity by effective width approach

$M_{EW} = 1.91$ tm

The ratio of the capacities

$$\frac{M_{EW}}{M_{SR}} = \underline{1.138}$$

The ratio $\frac{M_{EW}}{M_{SR}}$ has been evaluated for various sections in a similar way and are tabulated in Table 2.1

In the first three cases the (w/t) ratio of the unstiffened element was kept very near the $(w/t)_{lim}$ value whereas in the succeeding ones the (w/t) of the element was kept at a much higher value.

2.6 Comparison for Axially Loaded Members:

Although finding the axial load capacity of a cold-formed member in compression is not a trial and error problem, again the semi-empirical formulae for effective widths make it difficult to do algebraical manipulations.

So, here again the comparison is illustrated by working out numerically the capacities of sections.

The parameters which are more likely to affect the comparison are the (w/t) ratio of the unstiffened elements and the slenderness ratio (KL/r) of the member as a whole.

I-sections made out of two channels with unstiffened equal flanges are considered. The reason behind this choice is the fact that these sections can undergo only flexural buckling and not torsional flexural buckling. The notations for section dimensions are shown in Fig. 2.3. The member is restrained against buckling in the plane of the web.

h - depth of the section

a - ratio of the width of one unstiffened element to depth.

A model calculation is shown below.

Let the effective length of the member be 150 cm.

The following section is chosen;

$h = 30$ cms ; $t = 0.25$ cms; $a = 0.25$

$\frac{h}{t} = 120$; $\frac{ah}{t} = 30$.

The radius of gyration of the unreduced section

$$r = \sqrt{I/A} = h\sqrt{\frac{(a+1/6)}{(4a+2)}} = 13.42 \text{ cms}$$

(a) SR Approach:

Allowable stress for flanges $f_a = 790$ kg/cm^2

Stress form factor $Q_s = \frac{790}{1450} = 0.545.$

Evaluation of Q_a(area form factor);

$$\frac{h_e}{t} = \frac{2120}{\sqrt{f}}\left(1 - \frac{465}{(w/t)\sqrt{f}}\right) = 65.03$$

where, 'h_e' is the effective width of the web. The stress value 'f' is 790 kg/cm^2.

$$Q_a = \frac{\text{Total effective area}}{\text{Gross area}} = \frac{2h_e + 4ah}{2h + 4\,ah} = 0.69$$

Form factor $Q = Q_a \times Q_s = 0.379$

$$\frac{C_c}{\sqrt{Q}} = \sqrt{\frac{2\pi^2 E}{Q\,F_y}} = 216.$$

The value of E is taken as 2074000 kg/cm^2.

Since (KL/r) is less than $\frac{C_c}{\sqrt{Q}}$ the average allowable stress

$$F_{all} = 0.522\,Q\,F_y - \left(\frac{QF_y \times \frac{KL}{r}}{12500}\right)^2 \quad \text{(Sec. 6.1.1.a of Ref.(1))}$$

$$= 458 \text{ kg/cm}^2$$

Axial load capacity P_{SR} = 10.3 tons .

(b) EW Approach:

The effective width of flanges is to be found using equation (2.4) for a stress f = 1450 kg/cm^2.

$$\frac{a_e h}{t} = 18.74 \quad a_e\,h = 4.68 \text{ cms}$$

$\frac{h_e}{t}$ is to be found for stress f = 1450 kg/cm^2

$$\frac{h_e}{t} = 50.00.$$

$$Q = 0.456$$

$$\frac{C_c}{\sqrt{Q}} = 190$$

Hence $F_{all} = 587\ kg/cm^2$

Axial load capacity $P_{EW} = 13.21$ tons

Ratio of the capacities $\frac{P_{EW}}{P_{SR}} = 1.28$.

The ratio $\frac{P_{EW}}{P_{SR}}$ has been evaluated for various sections in a similar way and are tabulated in Table 2.2.

The first two cases have a small (KL/r) ratios whereas the next two have a high value in the viscinity of ,but less than $C_c/\sqrt{Q}$. The last case has a (KL/r) ratio which is less than $C_c/\sqrt{Q}$ in the stress reduction approach and greater than $C_c/\sqrt{Q}$ in the effective width approach.

2.7 Discussions on Flexural Members:

Table 2.1 shows the value of M_{EW}/M_{SR} for the different sections considered. It is very obvious that the capacity is much greater when the effective width formula is used. Another important conclusion one can make is that the ratio M_{EW}/M_{SR} is large when the section has an unstiffened element with a large (w/t) ratio when compared to the ratio M_{EW}/M_{SR} when the section has an unstiffened element with a (w/t) ratio just larger than $(w/t)_{lim}$. It can also be concluded that for the same (w/t) ratio of the unstiffened element the ratio M_{EW}/M_{SR}

decreases as the neutral axis shifts towards the compression flange. It can also be noted that such a decrease is very minimal. The ratio of the top and bottom flange areas A_T/A_B indicates whether the neutral axis is closer to the compression or tension flange. But this ratio is not as good an indicator as y_t in predicting the increase in capacity since the area of the web is not taken into account.

The foregoing conclusions can be explained as follows:

(i) When the section has an unstiffened element with a large (w/t) ratio the stresses in both the flanges are reduced to the level allowable for the compression flange. This affects the web also. In the effective width approach the tension flange and the webs are stressed almost to their capacity.

The (w/t) ratios of elements of sections considered fall in the range where inelastic local buckling occurs in the unstiffened elements (Ref. Fig. C.6 of Ref.(5)). Sections with unstiffened elements with (w/t) large enough to undergo elastic local buckling will prove to be highly uneconomical irrespective of the method used to find the capacity and so they should be avoided as far as possible.

(ii) In the effective width approach, if y_t is greater than 0.5 and stays so even after buckling the compression flange is subjected to a lesser stress than $0.6 \times F_y$ and hence has

TABLE 2.1

COMPARISON FOR FLEXURAL MEMBERS

Sl. No.	Section dimensions				y_t^*	$\frac{A_T}{A_B}$ **	(w/t) of unstiffened flange	$\frac{M_{EW}}{M_{SR}}$
	t cms	h cms	a	c				
1	0.25	30	0.30	0.125	0.490	1.20	15	1.14
2	0.25	30	0.25	0.125	0.500	1.00	15	1.13
3	0.25	30	0.20	0.125	0.510	0.80	15	1.10
4	0.25	30	0.50	0.200	0.483	1.25	24	1.38
5	0.25	30	0.40	0.200	0.500	1.00	24	1.38
6	0.25	30	0.30	0.200	0.519	0.75	24	1.37

* - y_t computed on the basis of unreduced section

**- Ratio of top flange area to bottom flange area

TABLE 2.2

COMPARISON FOR COLUMN MEMBERS

Sl. No.	Section dimensions				(w/t) of unstiffened flange	$(\frac{A_U}{A_S})^*$	$(\frac{KL}{r})$	$\frac{P_{EW}}{P_{SR}}$
	t cms	h cms	a	KL cms				
	0.25	30	0.25	150	30.00	0.5	13.42	1.28
	0.25	30	0.10	150	12.00	0.2	15.00	1.03
	0.15	10	0.30	500	20.00	0.6	131.00	1.16
	0.15	10	0.20	500	13.33	0.4	138.00	1.07
	0.15	10	0.30	600	20.00	0.6	157.22	1.02

* - Ratio of area of unstiffened elements to stiffened elements

a higher effective width. If y_t shifts towards the tension flange ($y_t < 0.5$) after buckling , the compression flange is subjected to the maximum stress. The reduction in capacity in the former case is more.

If y_t is greater than 0.5, the capacity will be reduced even in the stress reduction approach. But the reduction will almost be insignificant because even if y_t is less than 0.5 the utilisation of the tension flange will not be significantly higher.

In the absence of experimental evidence the above explanation cannot be taken as conclusive. More numerical examples should be worked out and the trend of ratio M_{EW}/M_{SR} fixed before anything conclusive can be said of the same.

Limitations of the comparison of capacities obtained from the two approaches are enumerated in the following lines. The number of sections considered are not many and hence the trend of the ratio M_{EW}/M_{SR} could not be fixed conclusively with respect to some of the parameters viz. the position of the neutral axis of the unreduced cross section and the ratio of the areas of the tension flange to the compression flange.

The open hat section is very stable against lateral buckling. One other section used commonly for beam members is the doubly symmetrical I-section which is not very stable against lateral buckling. For laterally unbraced I - beams

with the span to height (l/h) ratio in the practical range of 15 to 20, it can be shown that the flange width (designated 'ah' in Fig. 2.3) should be about 0.8 times the height 'h' for no stress reduction. This means the section will have an unstiffened element with a (w/t) ratio much in excess of what is desirable from an economic point of view. This should be avoided by bracing. It is in accordance with good practice if the bracings are spaced in such a way that stress is not reduced due to lateral buckling. In this case there is not much difference between I-section and the open hat section since any change in the disposition of the effective width of flanges parallel to the axis of bending will not affect the capacity of the section. So, for I-beams restrained or safe against lateral buckling the same trend of the ratio M_{EW}/M_{SR} obtained for hat sections can be taken to hold good.

For section which are specialities of cold-formed construction such comparison should be made individually.

2.8 Discussion on Compression Members:

Refering to Table 2.2 , here again it is seen that use of effective width equation for unstiffened elements results in a better utilization of a section.

The following conclusions can be made with regard to the effects of various section and member parameters:

(i) As in flexural members, here also the ratio of the capacities P_{EW}/P_{SR} increases with increase in the (w/t) of the unstiffened element.

(ii) The (KL/r) ratio of the member as a whole doesnot affect the ratio as far as it lies below $C_c/\sqrt{Q}$ in both the approaches.

(iii) The result of case number 5 shows an astonishingly low value of P_{EW}/P_{SR} even for a large (w/t) of unstiffened elements. In this case the (KL/r) of the member is less than $C_c/\sqrt{Q}$ in the stress reduction approach and hence the Structural Stability Research Council Formula for F_{all} (i.e.)

$$F_{all} = 0.522\ QF_y - \left(\frac{QF_y\ KL/r}{12500}\right)^2 \quad \text{should be used.}$$

The (KL/r) of the member is greater than $C_c/\sqrt{Q}$ in the effective width approach making the formula

$$F_{all} = \frac{10680000}{(KL/r)^2} \quad \text{applicable.}$$

This reduces the capacity of the section in the effective width approach bringing down the value of P_{EW}/P_{SR}.

(iv) When (KL/r) is greater than $C_c/\sqrt{Q}$ the local buckling of the elements does not figure in the allowable stress calculation. This is because the column will be so slender that overall buckling will occur before its elements undergo any local buckling. Hence both the approaches will give identical capacity.

The limitation of the comparison made lies in the fact that only a small number of sections were considered. But still the trend of the ratio P_{EW}/P_{SR} could be fixed better than in the case of flexural members. Only I-sections were considered. Doubly symmetrical I-sections, point symmetrical Z-section cannot undergo torsional-flexural buckling. Any singly symmetrical section can buckle in torsional-flexural mode. In such cases the proper allowable average stress should be considered in computing the capacities.

2.9 Deficiencies of Effective Width Approach:

The effective width approach takes into account the post buckling strength of unstiffened elements in a way similar to the one adopted for stiffened elements. But, there is a type of section which will show no post buckling strength. This is the angle section made up of only unstiffened elements. This is because the two components of the angle section will buckle in the same direction when the critical load is reached resulting in a twisting like distortion of the member. This leads to an early collapse. This fact is not taken into account in the effective width approach. This results in a unsafe design. The effective width formula should be modified for elements of angle sections only.

In the stress reduction approach the same has been taken into account by prescribing, for the same (w/t) ratio,

a smaller allowable stress for elements of angle sections than of other sections.

2.10 General Conclusion:

It is found that the effective width approach yields a higher capacity under both axial force and moments in most of the situations. It is also to be noted that even in unstiffened elements with large (w/t) of the order of 58, the out-of-plane deformations in the post buckling stage is of the order of 2.5 times the thickness of the element (2).

There is a distinct advantage of using the effective width approach for unstiffened elements. The (w/t) of stiffened flanges , stiffened by a simple lip is restricted to 60 because the requirement for the lip to be a stiffener leads to the value of (w/t) of lip to 10 when (w/t) of stiffened element is 60 . Any larger (w/t) ratio of stiffened element will lead to (w/t) of lip in excess of 10 resulting in stress reduction in the stiffened elements also. This restriction can obviously be done away with if the effective width approach is adopted for unstiffened flanges.

Hence it can be concluded that effective width approach be employed for unstiffened elements also.

CHAPTER 3

TWO SPAN CONTINUOUS BEAMS

3.1 General:

The present day design practice of choosing a section for the maximum bending moment irrespective of whether the sign of the bending moment changes sign along the length or not might lead to an unsafe design. It was pointed out in Chapter 1 that the algorithm developed by Gjelsvik and Bleustein (3) can not be successfully used in design problems. A modified version thereof has been developed and presented in this chapter. A design example is worked out to show the effectiveness of the method.

3.2 Analysis of Two Span Continuous Beams:

3.2.1 Basics:

The continuous-beam can be considered to be a beam with variable cross-sectional properties. The cross-sectional properties such as the position of neutral axis, the moment of inertia can be taken to be different in the different segments of the beam in which the moment does not change sign. For each of these segments, the cross-sectional properties can be taken to be constant and these properties can be fixed on the basis of the maximum bending moment

occuring in that segment. So, the whole beam can be considered to be made up of smaller segments in each of which the properties are constant. At the junction of two elements the moment will be zero. The conditions of compatibility at the junctions of the elements can be employed to find out the constants appearing in the expressions for slope and deflection of the beam.

It shall be noted however that the boundaries of the elements are not known apriori and they depend on the ratios of the moments of inertia of the different segments. This suggests that only a procedure of successive approximation can lead to the solution of the problem.

Fig. (3.1) shows a continuous beam with unequal spans and subjected to a uniformly distributed load of intensity 'q'. The longer span is always considered as span 1. Fig. (3.2) shows the different segments when the deflection curve is such that there are two inflection points, one in each span. It also shows the bending moment diagram and the deflected shape of the beam qualitatively. Fig. (3.3) shows the same things when the beam deflects in such a way that there is only one point of inflection . Obviously this point is in the longer span. The ratio of the span will be the important parameter which decides the type of deflection which would occur. It is obvious that the second type of deflection would

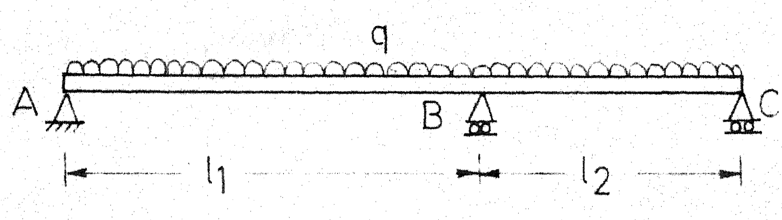

FIG.3.1 CONTINUOUS BEAM

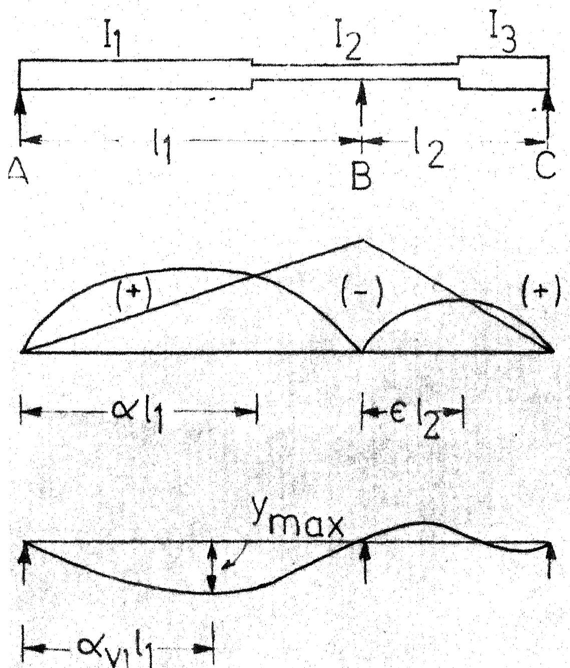

FIG.3.2 FIGURES FOR CASE I

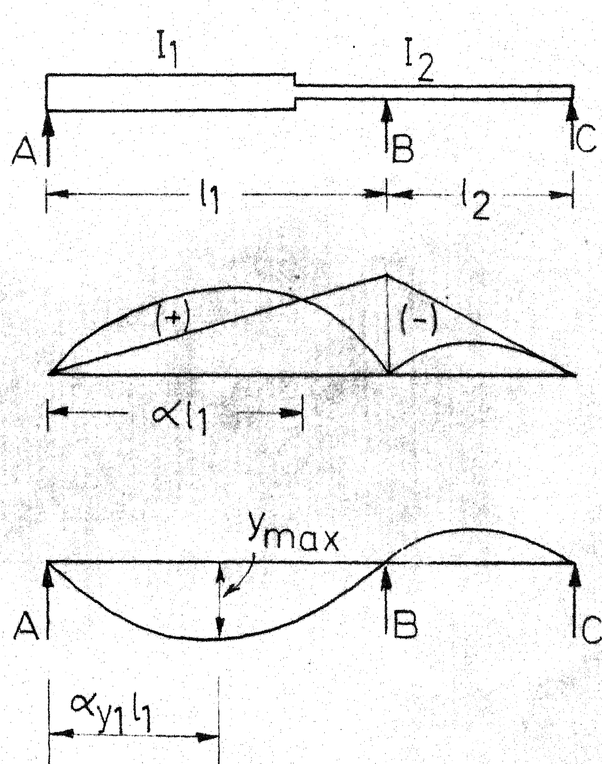

FIG.3.3 FIGURES FOR CASE 2

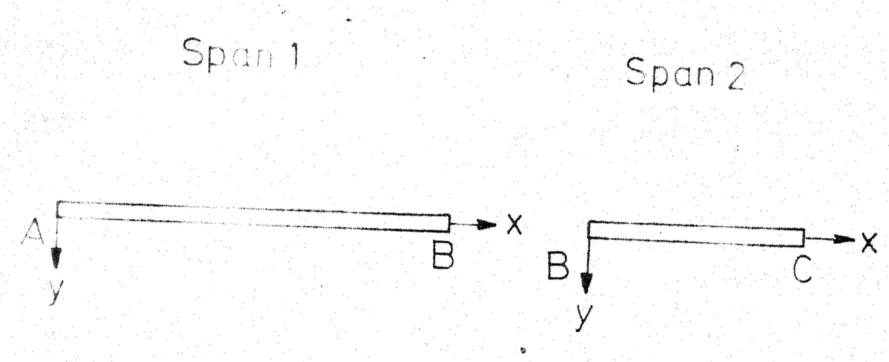

FIG 3·4 CO-ORDINATE AXES

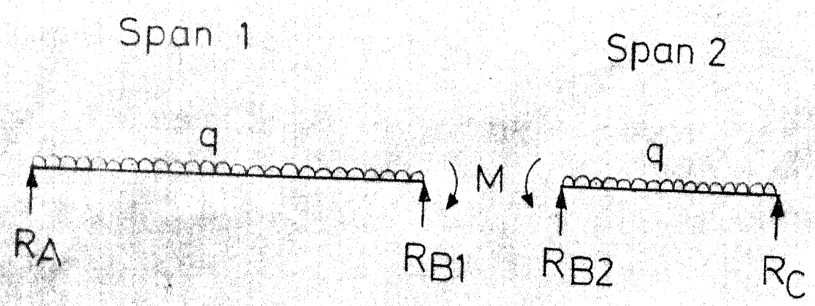

FIG. 3·5 FREE BODY DIAGRAM

occur for large ratios of l_1/l_2.

The situation shown in Fig. (3.2) shall henceforth be called Case I and the one shown in Fig. (3.3) , Case II. The moments of inertia of the segments shown are indicated in Figs. (3.2) and (3.3) . The point of maximum deflection in span 1 is located at a distance $\alpha_{y1}l_1$ from the support A. This will be required at the end of the design procedure to check for the restriction on the deflection. The other notations used in the figures are self explanatory. Fig. (3.4) shows the coordinate axes considered for the different portions of the beam. The moment producing tension at the bottom fibre is considered positive.

3.2.2 Derivation of Governing Equations for Case I:

With the foregoing convention for bending moment, the basic equation can be written as

$$EI \frac{d^2 y}{d x^2} = - M(x) \qquad (3.1)$$

Where,

E - Young's modulus

I - Moment of inertia

M(x)-Bending moment

The free body diagram of span 1 and span 2 isolated are shown in Fig. (3.5) . 'M' indicates the negative bending moment over the support B.

Considering span 1;

For $0 \leq x \leq \alpha l_1$ (Fig.(3.2))

$$EI_1 \frac{d^2y}{dx^2} = -\left(\frac{ql_1}{2} - \frac{M}{l_1}\right) x + \frac{qx^2}{2}$$

On integration

$$EI_1 \frac{dy}{dx} = -\left(\frac{ql_1}{2} - \frac{M}{l_1}\right) \frac{x^2}{2} + \frac{qx^3}{6} + C_1$$

Integrating once again

$$EI_1 y = -\left(\frac{ql_1}{2} - \frac{M}{l_1}\right) \frac{x^3}{6} + \frac{qx^4}{24} + C_1 x + C_2$$

For $\alpha l_1 \leq x \leq l_1$ (Fig. (3.2))

$$EI_2 \frac{d^2y}{dx^2} = -\left(\frac{ql_1}{2} - \frac{M}{l_1}\right) x + \frac{qx^2}{2}$$

On integration

$$EI_2 \frac{dy}{dx} = -\left(\frac{ql_1}{2} - \frac{M}{l_1}\right) \frac{x^2}{2} + \frac{qx^3}{6} + C_3$$

Integrating again

$$EI_2 y = -\left(\frac{ql_1}{2} - \frac{M}{l_1}\right) \frac{x^3}{6} + \frac{qx^4}{24} + C_3 x + C_4$$

The constants of integration C_1, C_2 , C_3 and C_4 are evaluated by using the following conditions:

$$y\ (x = 0) = 0$$

$$y\ (x = l_1) = 0$$

and the compatibility conditions at $x = \alpha l_1$ that the deflection and the slope are same as computed from the two regions of span 1. The equation for the deflection in the region $0 \le x \le \alpha l_1$ is given below in its final form:

$$EI_1 y = -\left(\frac{ql_1}{2} - \frac{M}{l_1}\right)\frac{x^3}{6} + \frac{qx^4}{24} + C_1 x \qquad (3.2)$$

Where,

$$C_1 = \left[\left(\frac{ql_1}{2} - \frac{M}{l_1}\right)\frac{\alpha^2 l_1^2}{2} - \frac{q\alpha^3 l_1^3}{6}\right](1-R_1) + \frac{R_1 q l_1^3}{24}$$

$$- \frac{R_1 M l_1}{6} - (1-R_1)\left[\left(\frac{ql_1}{2} - \frac{M}{l_1}\right)\frac{\alpha^3 l_1^2}{3} - \frac{q\alpha^4 l_1^3}{8}\right].$$

Considering span 2 ;

$0 \le x \le \varepsilon l_2$ (Fig. (3.2))

$$EI_2 \frac{d^2y}{dx^2} = -\left(\frac{ql_2}{2} + \frac{M}{l_2}\right) x + M + \frac{qx^2}{2}$$

On integration

$$EI_2 \frac{dy}{dx} = -\left(\frac{ql_2}{2} + \frac{M}{l_2}\right)\frac{x^2}{2} + Mx + \frac{qx^3}{6} + C_5$$

Again integrating

$$EI_2 y = -\left(\frac{ql_2}{2} + \frac{M}{l_2}\right)\frac{x^3}{6} + \frac{Mx^2}{2} + \frac{qx^4}{24} + C_5 x + C_6$$

For $\varepsilon l_2 \le x \le l_2$ (Fig.(3.2))

$$EI_3 \frac{d^2y}{dx^2} = -\left(\frac{ql_2}{2} + \frac{M}{l_2}\right) x + M + \frac{qx^2}{2}$$

On integration

$$EI_3 \frac{dy}{dx} = -\left(\frac{ql_2}{2} + \frac{M}{l_2}\right)\frac{x^2}{2} + Mx + \frac{qx^3}{6} + C_7$$

On integration

$$EI_3 y = -\left(\frac{ql_2}{2} + \frac{M}{l_2}\right)\frac{x^3}{6} + \frac{Mx^2}{2} + \frac{qx^4}{24} + C_7 x + C_8$$

The constants C_5, C_6, C_7 and C_8 are evaluated by using the following conditions:

$$y\,(x = 0) = 0$$

$$y\,(x = l_2) = 0$$

and the compatibility conditions that at $x=\varepsilon l_2$, the deflection and the slope are same as computed from the two regions of span 2.

The only other unknown in the foregoing sets of equations is the end moment 'M'. This is evaluated using the condition that the slope is same over the support B as computed from both the spans. Equation (3.3) gives the expression for 'M' implicitly.

$$M\left[\frac{l_1}{3}\left\{\frac{(1-R_1)\alpha^3}{R_1} + 1\right\} + l_2\left\{\frac{(1-R_2)\varepsilon^3}{3} - (1-R_2)\varepsilon^2 + (1-R_2)\varepsilon + \frac{R_2}{3}\right\}\right]$$
$$= q\left[l_1^3\left\{\frac{1}{24} + \frac{(1-R_1)\alpha^3}{6R_1} - \frac{(1-R_1)}{R_1}\frac{\alpha^4}{8}\right\} + l_2^3\left\{\frac{(1-R_2)\varepsilon^4}{8}\right.\right.$$
$$\left.\left. - \frac{(1-R_2)\varepsilon^3}{3} + \frac{(1-R_2)\varepsilon^2}{4} + \frac{R_2}{24}\right\}\right] \qquad (3.3)$$

Where,

$$R_1 = I_1/I_2 \;;\quad R_2 = I_2 / I_3 .$$

The condition that $EI_1 \dfrac{d^2y}{dx^2} = 0$ at $x = \alpha l_1$ gives

$$M = \frac{ql_1^2(1-\alpha)}{2} \tag{3.4}$$

Similarly the condition that $E\,I_2 \dfrac{d^2y}{dx^2} = 0$ at $x = \varepsilon l_2$ gives

$$M = \frac{ql_2^2\varepsilon}{2} \tag{3.5}$$

Comparing equations (3.4) and (3.5) a relation between α and ε can be established as

$$\frac{\varepsilon}{(1-\alpha)} = \frac{l_1^2}{l_2^2} \tag{3.6}$$

Substituting for 'M' in equation (3.3) from equation (3.4) the required polynomial in α, the solution of which will give α for any combination of $\dfrac{l_1}{l_2}$, R_1, R_2 is obtained. This equation is given as follows:

$$\alpha^4\left[\frac{\beta^5(1-R_2)}{24} - \frac{(1-R_1)}{24\ R_1}\right] + \alpha^3\left[\frac{\beta^3(1-R_2)}{6} - \frac{\beta^5(1-R_2)}{6}\right]$$
$$+\ \alpha^2\left[\frac{\beta^5(1-R_2)}{4} - \frac{\beta^3(1-R_2)}{2} + \frac{\beta(1-R_2)}{4}\right] + \alpha\left[\frac{\beta^3(1-R_2)}{2}\right.$$
$$\left. - \frac{\beta^5(1-R_2)}{6} - \frac{\beta(1-R_2)}{2} - \frac{R_2}{6\beta} - \frac{1}{6}\right] + \left[\frac{1}{8} + \frac{\beta(1-R_2)}{4}\right.$$
$$\left. - \frac{\beta^3(1-R_2)}{6} + \frac{\beta^5(1-R_2)}{24} + \frac{R_2}{6\beta} - \frac{\beta^3 R_2}{24}\right] = 0 \tag{3.7}$$

where,

$$\beta = l_1/l_2\ .$$

Let $x = \alpha_{y1} l_1$ be the point where the maximum deflection occurs in span 1. Obviously this point will lie between x = 0 and $x = \alpha l'_1$.

Using the condition that $EI_1 \frac{dy}{dx} = 0$ at $x = \alpha_{y1} l_1$ the following equation is obtained.

$$\frac{\alpha_{y1}^3}{6} - \frac{\alpha_{y1}^2}{4} + \frac{(1-R_1)\alpha^3}{12} - \frac{(1-R_1)\alpha^4}{24} - \frac{R_1}{24} + \frac{R_1\alpha}{12} = 0 \qquad (3.8)$$

The root α_{y1} of the above equation fixes the point of maximum deflection in span 1.

3.2.3 Derivation of Governing Equations for Case II:

Fig. (3.5) gives the free body diagram of the two spans isolated. As can be seen from Fig. (3.3), R_2 does not exist in this case.

Considering span 1 ;

A comparison of Figs. (3.3) and (3.2) clearly indicates that the behaviour of span 1 is identically same in both the cases. Therefore, the equation for deflection obtained by integrating the equation (3.1) twice will be identical to what has been obtained for Case I.

Considering span 2;

For $0 \leq x \leq l_2$ (Fig. 3.3)

$$EI_2 \frac{d^2y}{dx^2} = -\left(\frac{ql_2}{2} + \frac{M}{l_2}\right) x + M + \frac{qx^2}{2}$$

On integration

$$EI_2 \frac{dy}{dx} = -\left(\frac{ql_2}{2} + \frac{M}{l_2}\right) \frac{x^2}{2} + Mx + \frac{qx^3}{6} + C_9$$

Integrating once again

$$EI_2 y = -\left(\frac{ql_2}{2} + \frac{M}{l_2}\right) \frac{x^3}{6} + \frac{Mx^2}{2} + \frac{qx^4}{24} + C_9 x + C_{10}$$

The constants C_9 and C_{10} are evaluated using the following boundary conditions:

$$y\ (x = 0) = 0$$

$$y\ (x = l_2) = 0$$

To evaluate 'M', the condition that slope is same over the support B as computed from the two spans is used. The following equation, implicit in 'M', is obtained as

$$-\left[\left(\frac{ql_1}{2} - \frac{M}{l_1}\right) \frac{l_1^2}{2} - \frac{ql_1^{\,3}}{6}\right] + \frac{ql_1^3}{24} - \frac{Ml_1}{6} - \frac{(1-R_1)}{R_1}$$
$$\left[\left(\frac{ql_1}{2} - \frac{M}{l_1}\right) \frac{\alpha^3 l_1^2}{3} - \frac{q\alpha^4 l_1^3}{8}\right] = \frac{ql_2^3}{24} - \frac{Ml_2}{3} \qquad (3.9)$$

The condition that $EI \dfrac{d^2y}{dx^2} = 0$ at $x = \alpha l_1$ yields the following equation:

$$M = \frac{ql_1^2(1 - \alpha)}{2} \qquad (3.10)$$

Substituting for 'M' in equation (3.9) from equation (3.10) yields an equation similar to equation (3.7). The fourth order

polynominal in α so obtained is given as follows:

$$\frac{\alpha^4(1-R_1)}{R_1} + 4\alpha\left(1+\frac{1}{\beta}\right) + \frac{1}{\beta^3} - \frac{4}{\beta} - 3 = 0 \qquad (3.11)$$

The point of maximum deflection in span 1 is obtained in a simila way as was done for Case I and is given as

$$4\alpha_{y1}^3 - 6\alpha_{y1}^2\alpha + 2(1-R_1)\alpha^3 - R_1 + 2R_1\alpha - (1-R_1)\alpha^4 = 0 \qquad (3.12)$$

3.2.4 Solutions to Governing Equations:

The upper and lower limits of the variation of R_1 and R_2 are fixed as 0.80 and 1.20. Any value beyond these can be practically realised only if the section has elements of large (w/t) ratio. This will call for a change in the sectional dimensions from economy point of view. So, R_1 and R_2 are varied from 0.80 to 1.20 in steps of 0.05.

For a given combination of R_1, R_2 and β, if equation (3.7) does not give a value of α between 0 and 1 or the value of ϵ between 0 and 1, it can be concluded that Case II type of deflection occurs. It can also be concluded that Case II type of deflection will occur for the particular combination of R_1 and β for all values of R_2'. In this case equation (3.11) has to be solved to get the proper value of α. Figs. (3.6) through (3.10) show

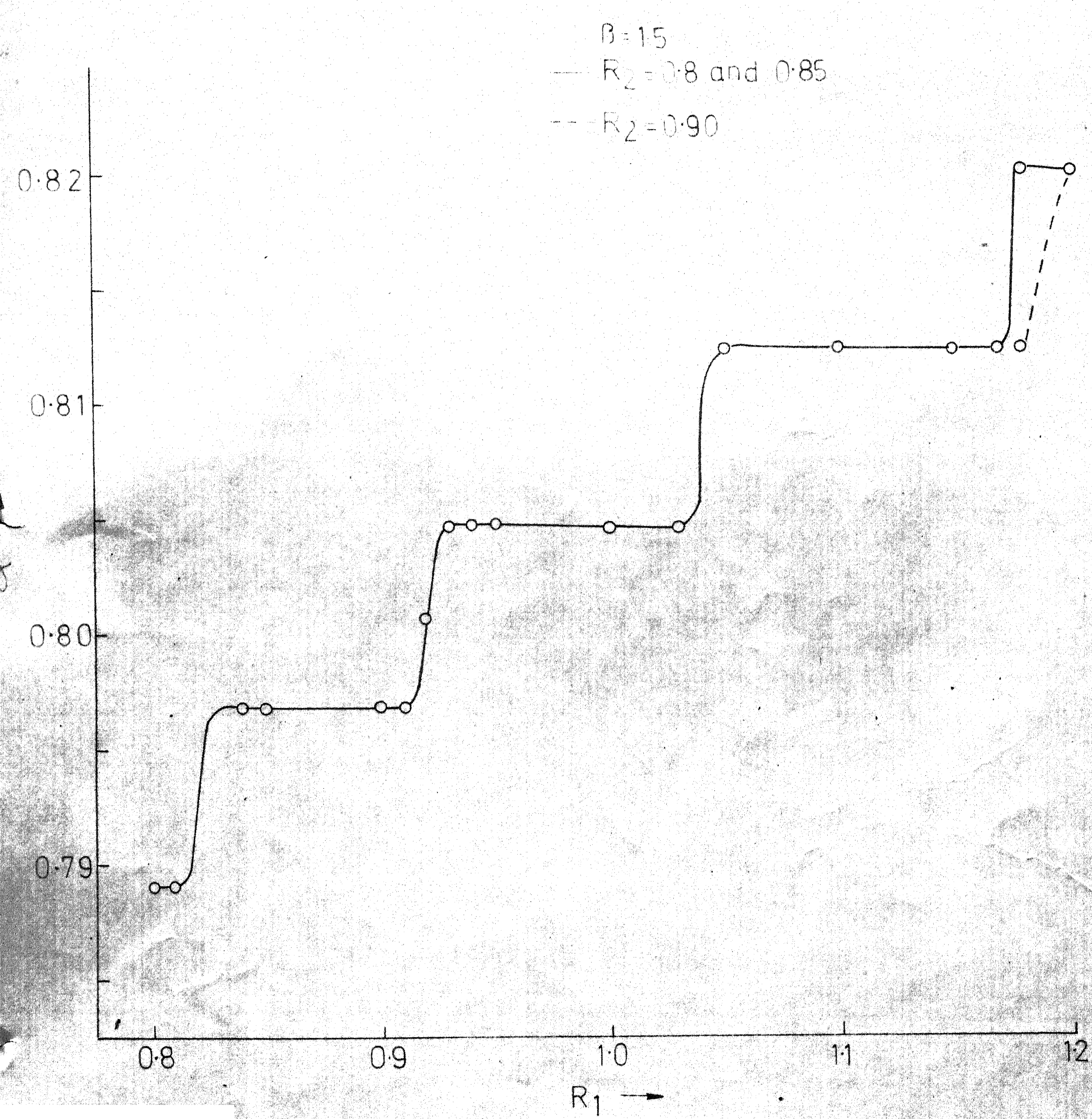

FIG. 3.6 DESIGN CHARTS FOR α

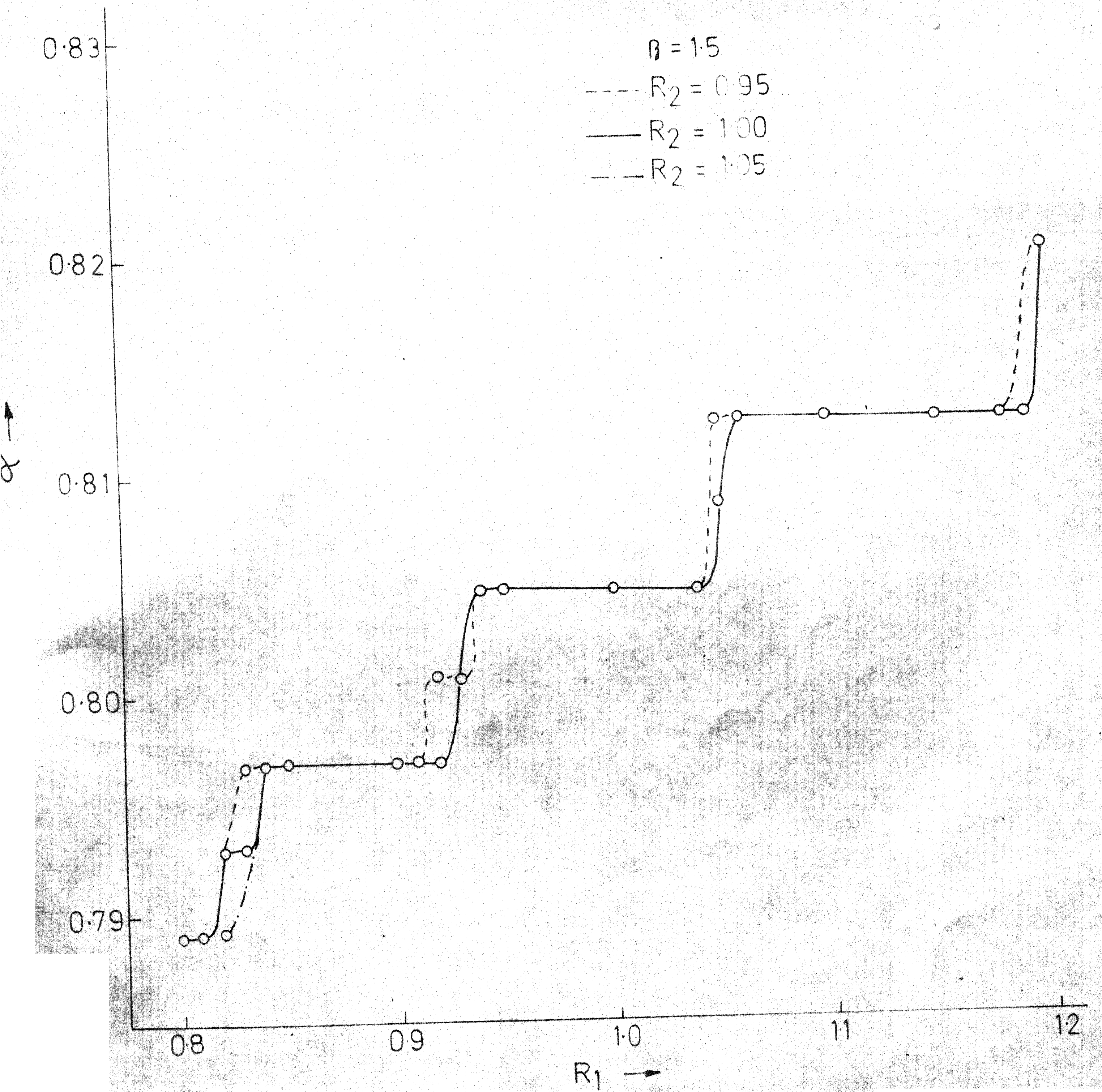

FIG. 3·7 DESIGN CHARTS FOR α

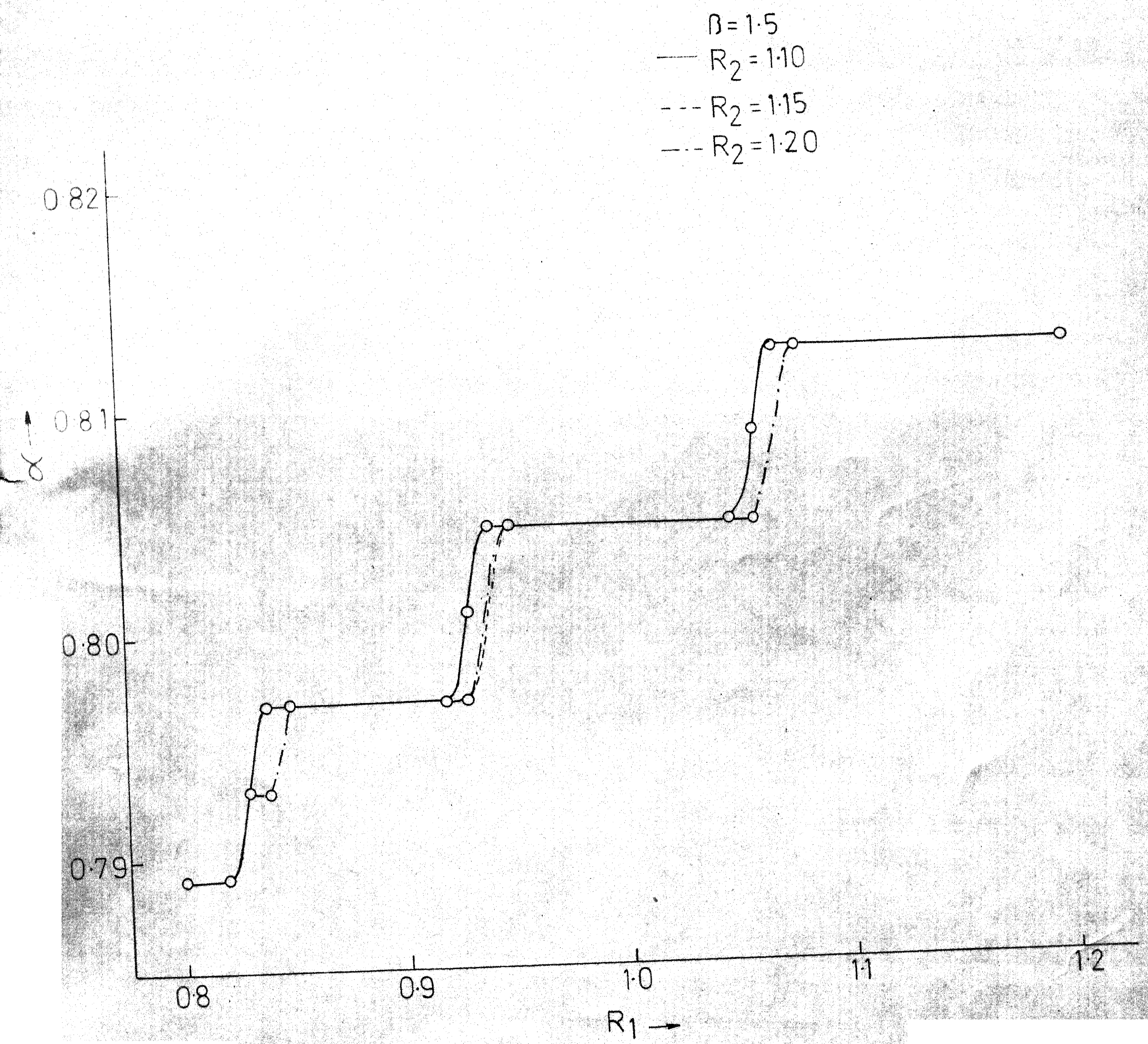

FIG. 3·8 DESIGN CHARTS FOR α

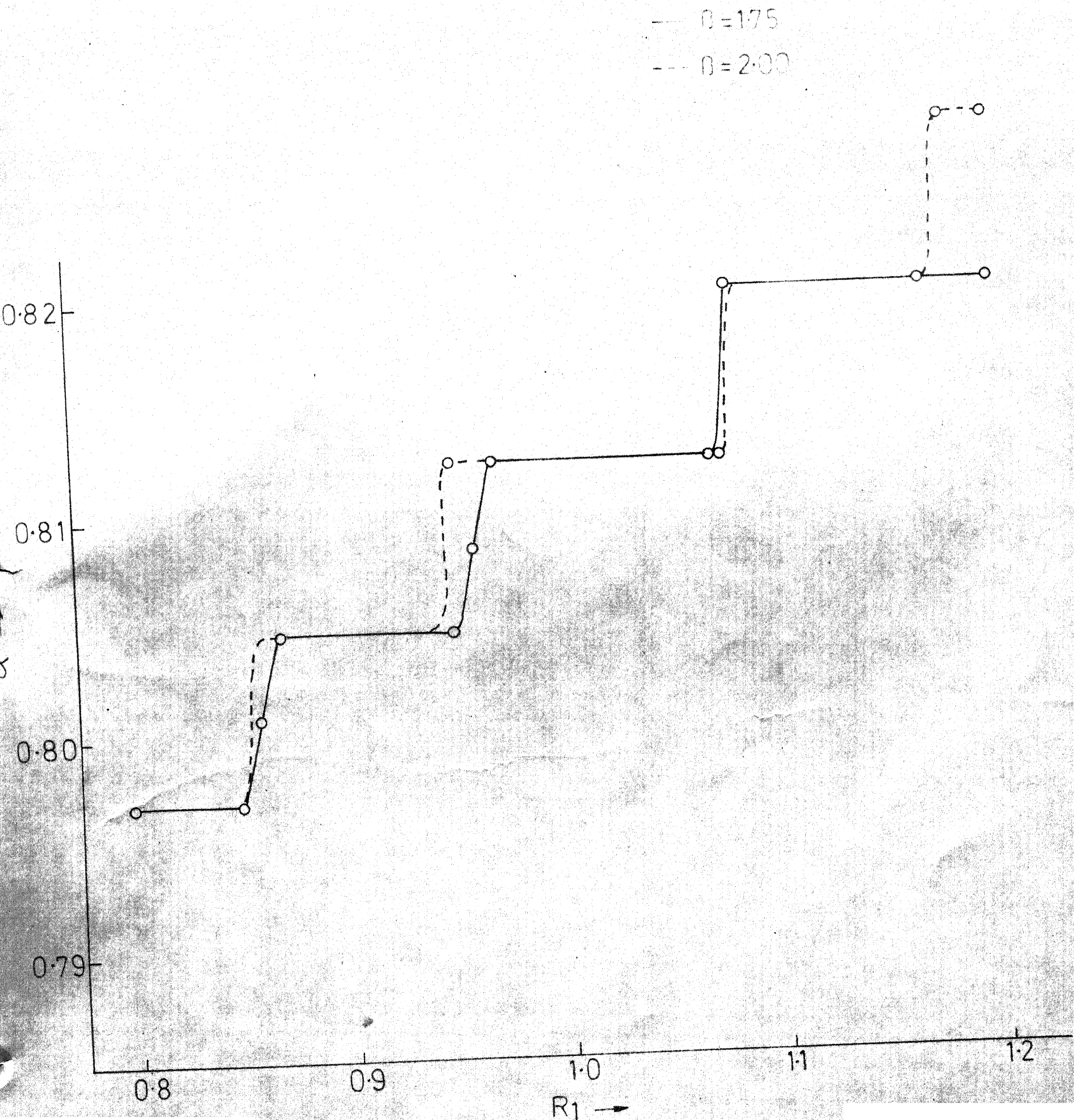

FIG. 3·9 DESIGN CHARTS FOR α

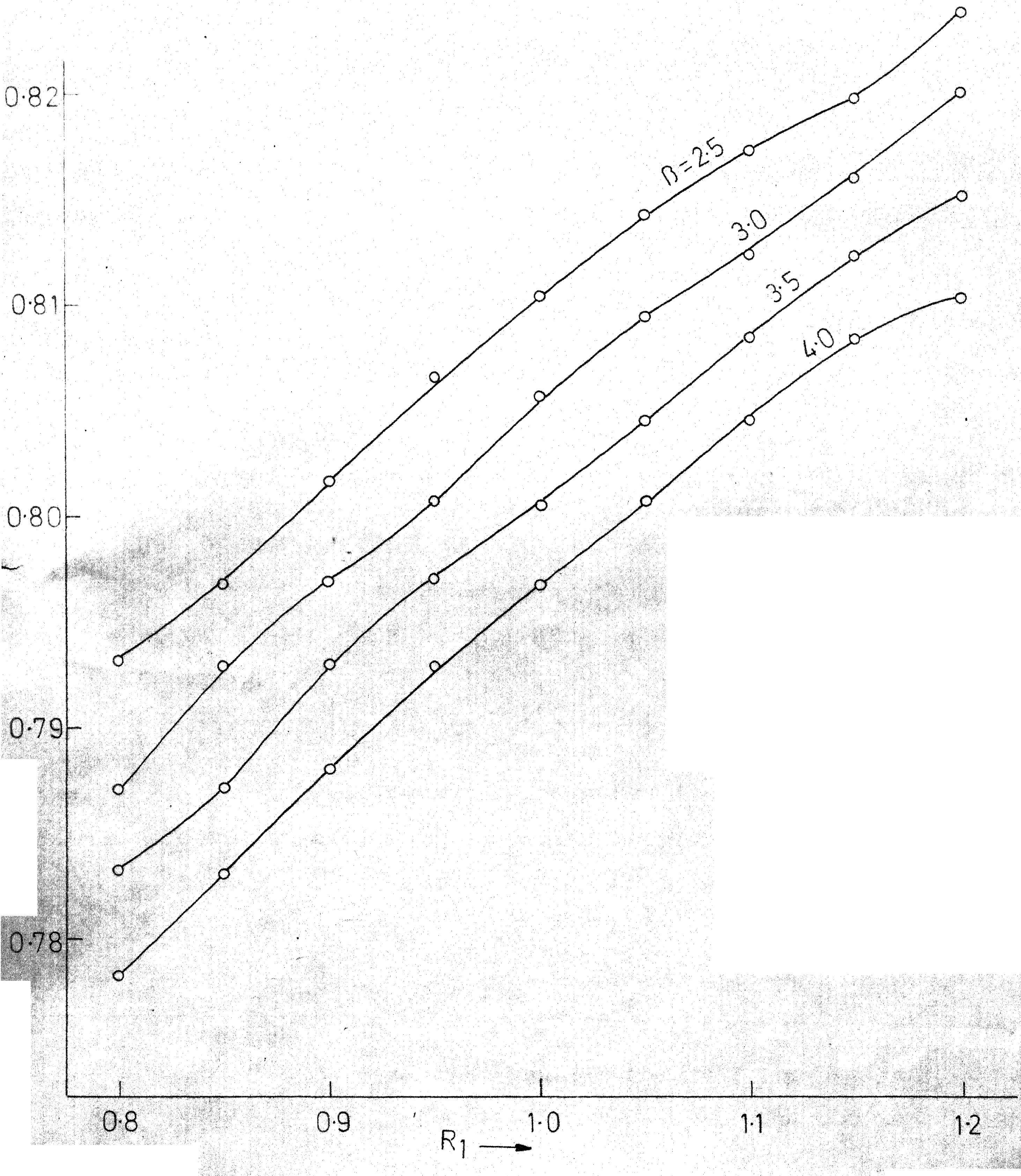

FIG. 3·10 DESIGN CHARTS FOR α

the variation of α with various combinations of R_1, R_2 , β or R_1, β as the case may be. These figures are to be used as design charts.

It is always necessary to ensure that the deflections do not exceed a prescribed limit. Here, the limit is taken as 1/325, where 'l' is the span. In general, cold-formed members will satisfy the deflection criteria because the flanges will be at a farther distance from the neutral axis than in hot rolled sections.

When the deflection has to be checked, the root α_{y1} of equations (3.8) or (3.12) depending on the situation has to be found. As the equations are third order polynominals, one can find the root without going in for any numerical (iterative) methods (6).

3.3 Design Algorithm:

It is assumed that spans and the loads are the design parameters. The aim is to design a section for the given parameters taking into account the variable cross sectional property.

The design shall be done following the steps given below:

(i) Given the spans and the loads , an analysis is performed assuming R_1=1.00 and R_2 = 1.00. The maximum

negative bending moment, M^- (over the intermediate support , at $x = l_1$ in span 1), the maximum positive bending moment in span 1, M_1^+ (occuring at $x = \frac{\alpha l_1}{2}$) and the maximum positive bending moment (applicable only if Case 1 type of deflection occurs) in span 2, M_2^+ (occuring at $x = \frac{(\varepsilon+1)l_2}{2}$) can be found out from the following equations.

$$
\left.
\begin{aligned}
M^- = M &= \frac{q l_1^2 (1-\alpha)}{2} \\
M_1^+ &= \frac{q\alpha^2 l_1^2}{8} \\
M_2^+ &= \frac{q(1-\varepsilon)^2 l_2^2}{8}
\end{aligned}
\right\} \qquad (3.13)
$$

The value of α can be read out from the corresponding graph.

(ii) The maximum moment of inertia of a section required to resist the moments found out in step 1 is calculated. The value thus obtained is increased by 15 to 20 percent and this increased value is taken as the moment of inertia 'I' of the cold-formed section.

(iii) Taking the depth of the beam in the range of $\frac{l_1}{15}$ to $\frac{l_1}{20}$, the section dimensions can be fixed. The shape of the section should have been decided on earlier. If there is an unstiffened flange the allowable stress for the same is calculated and if there is a stiffened flange, the maximum stress under which the element would be fully effective is found out using section 6.2 and 5.2.1.1 of Ref.(1).

(iv) The stresses in the compression flanges are found out.

$$f_1^c = \frac{M_1^+}{Z_c} \; ; \qquad f_2^c = \frac{M^-}{Z_c} \; ; \qquad f_3^c = \frac{M_2^+}{Z_c} \qquad (3.14)$$

Where,

f_1^c - stress in the compression flange under the maximum positive bending moment in span 1,

f_2^c - stress in the compression flange under the maximum negative bending moment over support B,

f_3^c - stress in the compression flange under the maximum positive bending moment in span 2 (applicable only if Case I type of deflection occurs),

Z_c - corresponding compression section modulus.

(v) If the stresses found out in step (iv) exceed the limiting stresses calculated in step (iii), it means the flange elements have locally buckled. The moments of inertia I_1, I_2, I_3 are fixed for the stresses f_1^c , f_2^c , f_3^c found out in step(iv) and the values of R_1 and R_2 calculated.

(vi) Step (i) is repeated with the values of R_1, R_2 found in step (v). Skipping steps (ii) and (iii) steps (iv) and (v) are performed.

This iterative procedure is continued till the stresses f_1^c , f_2^c, f_3^c obtained from step (iv) do not change significantly in successive iterations. At the end, the deflection should be estimated and checked to see if it is within the prescribed limits. The distance $\alpha_{y1} l_1$ where maximum deflection occurs is found out from equation (3.8) or (3.12) and the deflection from equation (3.2).

Figs. (3.6) through (3.9) show the variation of α with R_1 , R_2 and β for R_1 = 0.80 to 1.20 in steps of 0.01 , R_2= 0.80 to 1.20 in steps of 0.05 and β = 1.50 to 2.00 in steps of 0.25. It is seen that for all values of β considered above,Case I type of deflection occurs. In a design problem, for intermediate values of β between 1.50 and 2.00, a conservative design section can be arrived at by considering the maximum value of α in that range for M_1^+ and M_2^+ and the minimum value of α in that range for M^-.

For β values equal to or greater than 2.50 Case II type of deflection occurs. Here, for any intermediate value of β, α can be interpolated in that range and used. When β lies in between 2.00 and 2.50 the transition from Case I type of deflection to Case II type of deflection occurs. Use of design charts will be cumbersome in this range. So here again, a conservative design should be arrived at as explained in the previous paragraph.

3.4 Case of One Span Alone Loaded:

If the longer span alone is loaded, it can be shown that M_1^+ will be higher than what it will be if both the spans are loaded. Hence the section should be checked for this increased positive bending moment. In this case, irrespective of the parameters R_1, β only Case II type of deflection will occur.

The equations for finding out α and α_{y1} are given as follows:

$$\frac{\alpha^4(1-R_1)}{R_1} + 4\alpha\left(1+\frac{1}{\beta}\right) - \frac{4}{\beta} - 3 = 0 \qquad (3.15)$$

$$4\alpha_{y1}^3 - 6\,\alpha_{y1}^2\,\alpha + 2(1-R_1)\,\alpha^3 - R_1 + 2R_1\alpha - (1-R_1)\alpha^4 = 0 \qquad (3.16)$$

The equation (3.15) gives α and this fixes the maximum positive bending moment occuring at $x = \frac{\alpha l_1}{2}$ in span 1. Design charts can be made for this situation also. The root α_{y1} of equation (3.16) fixes the point of maximum deflection.

3.5 Design Example:

The design parameters are taken to be the spans and load intensity q . Taking $l_1 = 5.00$ m ; $l_2 = 3.33$ m, $q = 1$ t/m , then $\beta = 1.50$.

Step (i) Assuming $R_1 = 1.00$ and $R_2 = 1.00$

$\alpha = 0.8048$ (Fig. 3.7)

$M^- = 2.44$ tm ; $M_1^+ = 2.024$ tm; $M_2^+ = 0.436$ tm.

Step (ii) The moment of inertia required of the cross section

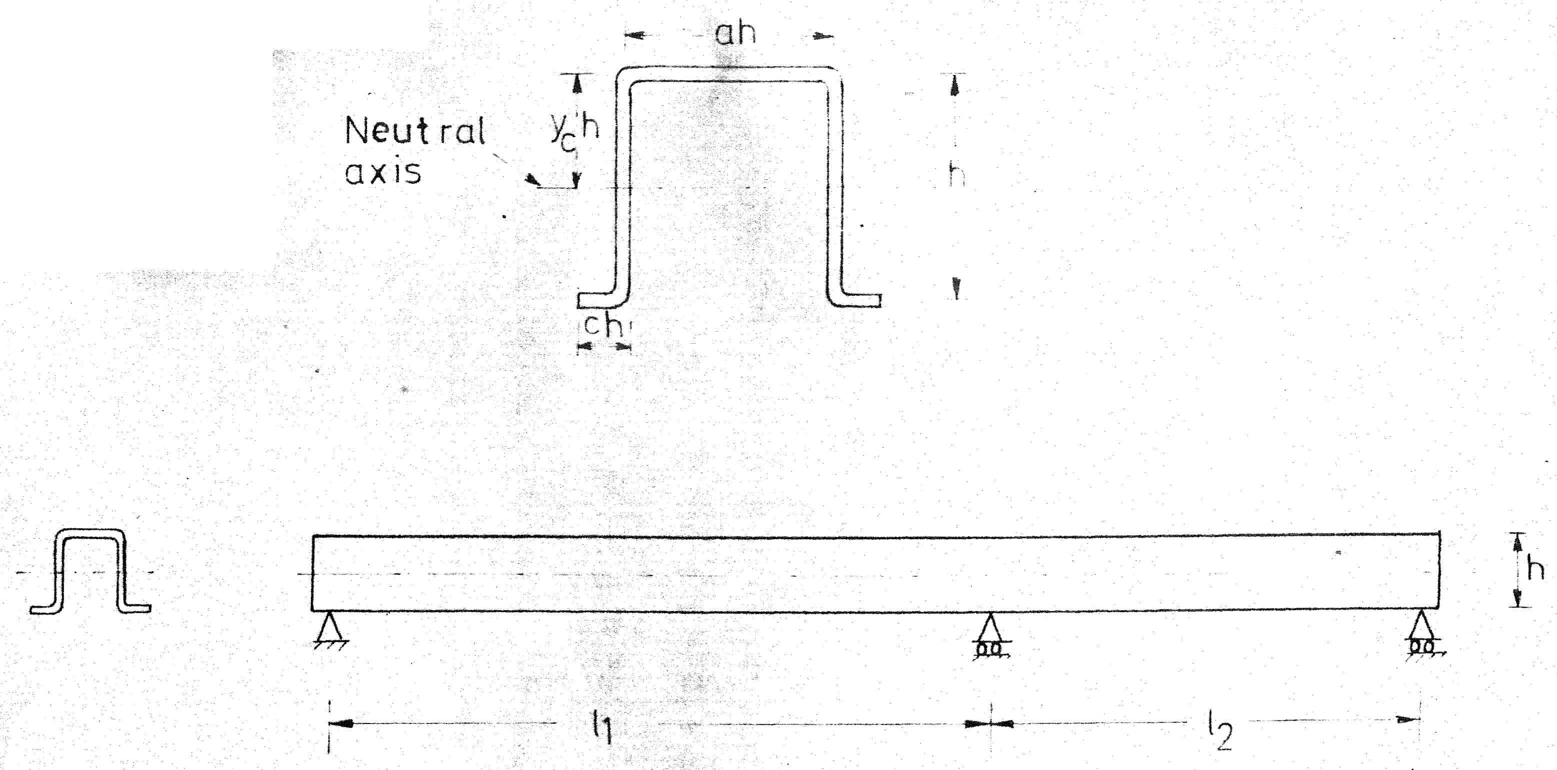

FIG. 3.11 CONTINUOUS BEAM (Design example)

$$I = \frac{244000 \times 16 \times 1.20}{1450}$$ (Assuming an overall depth of the beam as 32 cms)

$$= 3230 \text{ cm}^4$$

Step (iii) An open hat section is considered. The notations for the section are given in Fig. 3.11 . The dimensions of the sections are expressed as fractions of the depth 'h'.

a - Ratio of width of stiffened flange to the depth,

c - Ratio of width of one unstiffened flange to the depth

y_c - Ratio of the distance to stiffened flange from neutral axes to the depth

t - Thickness of the section.

The following trial section is chosen:

$h = 32$ cms ; $t = 0.3$ cm ; $a = 0.45$; $c = 0.12$

$y_c = 0.461$; $I = 3294 \text{ cm}^4$

The allowable stress in the unstiffened flange = 1329 kg/cm^2

The maximum stress upto which the stiffened flange is fully effective = 894 kg/cm^2.

Step (iv) $f_1^c = 906 \text{ kg/cm}^2$

$f_2^c = 1278 \text{ kg/cm}^2$

$f_3^c = 195 \text{ kg/cm}^2$

$$\frac{a_e h}{t} = 47.76 \ ; \ a_e = 0.448$$

$y_c = 0.461$

$I_1 = 3293 \text{ cm}^4$; $I_2 = 3294 \text{ cm}^4$; $I_3 = 3294 \text{ cm}^4$

$R_1 = 1.00$; $R_2 = 1.00$.

In this case no iteration is necessary since in the first trial itself the stresses obtained exceed the limiting stresses only marginally resulting in no reduction of the moment of inertia. The design has to be checked for shear stress, web crippling, web buckling. The section chosen satisfies all these. A very approximate and conservative estimation of the maximum deflection can be made by the following formula :

$$y_{max} = \frac{5}{384} \times \frac{q l_1^4}{EI}$$

$$= 1.19 \text{ cms} < \frac{l_1}{325}$$

Hence it can be concluded that the section satisfies the deflection criteria also.

If the initial section chosen consists of stiffened and unstiffened elements of large (w/t) ratio the rationality of the design approach could be well understood.

The spans and the load considered in the foregoing example are moderate. It is seen that though an ISLB 200 with a sectional area 25.27 cm^2 is sufficient to resist the maximum moment, it does not satisfy the deflection criteria. An ISLB 225 will satisfy the deflection criteria also. The latter has a cross-sectional area of 29.92 cm^2. The cross-sectional area of the cold formed section is 25.82 cm^2. The comparison of the cross-sectional areas shows that the cold-formed section is about 14 percent economical.

CHAPTER 4

AN OVERVIEW

4.1 Summary:

The work described here in aims at suggesting modifications that can be adopted in the design procedure for cold-formed light gauge steel members to arrive at a more rational design. The present design procedure is highly conservative under some circumstances. Under some other circumstances one might tend to arrive at an unsafe design.

As cold-rolled steel sheets are not produced in India, one is unable to do any experimental work in this topic. Moreover, the factors involved are highly complex and hence a purely theoretical approach is almost an impossibility. It shall be noted that most of the equations currently being used in the codes of practice are atleast semiempirical in nature. These equations, in general, were obtained from experimental work. So, most of the work described herein is based on experimental data available in the literature.

The post buckling strength of unstiffened elements is not taken into account in the present codes of practice because of concern regarding the out-of-plane deformations in the post buckling range. However, recent research (2) shows

that the out-of-plane deformations, though greater than what they are for stiffened element, should not cause any worry. On the basis of test results an effective width equation was arrived at for unstiffened elements. In this work the advantages of using the effective width equation for unstiffened elements are brought out. In general, it is seen that the utilisation of a section under axial load and under moment is better if the effective width approach is used. It is more so if the section consists chiefly of stiffened elements and has a large unstiffened element. These comparisons have been illustrated by numerical examples only.

If a beam member, a part of a statically indeterminate structure, is subjected to moments of both signs, it should be noted that moments of inertia can be taken to be constant only over the segment of the beam in which the moment is of the same sign. The boundaries of these segments are not known apriori and they inturn depend on the moments of inertia of the section. So, only a trial and error procedure can aolve this problem. The algorithm developed by Bleustein and Gjelsvik (3) has been extended to continuous beams of unequal spans and also modified to suit day to day design problems. Design charts are also given . A design example is worked out.

4.2 Salient Points to be Noted:

An unstiffened element acting as a stiffener to an stiffened element, say a web, has to satisfy certain conditions on its rigidity. Otherwise, it ceases to be a stiffener. The condition for the minimum rigidity required is stipulated in Section 5.2.2.1 of Ref.(1) for an unstiffened element when stress reduction approach is employed. In this approach the element undergoes out-of-plane deformations only because of the initial imperfections present.

When the concept of effective width is made use of for unstiffened elements in designing a section, these elements undergo out-of-plane deformations in the post buckling stage which will be excessive. This reduces the stiffness offered by the unstiffened element to the stiffened element. This results in the reduction of the value of the constant k in equation 2.1, which was taken to be 4.0. So, the effective width of the stiffened element calculated as per Section 5.2.1.1 of Ref.(1) might be higher than what is actually realized in practice. Use of the equations given in Section 5.2.1.1 of Ref.(1) is not rational for the reason given above regarding out-of-plane deformations.

It is a possibility that this is the reason why in Fig. 7(a) of Ref.2 (Fig. A.1 herein, Appendix A) the test result is much less than that predicted by using the effective

width equation for unstiffened element. It is to be noted that (w/t) of the unstiffened element is as much as 57.5 and the (KL/r) is as low as about 50. The form factor, Q, has a greater effect on the average allowable stress F_{all} in the lower ranges of (KL/r).

In these cases it will be better and easier to alter the effective width of the stiffened elements by a factor which can atmost be 1.0 and which will depend on the (w/t) ratio of the unstiffened element. This factor can best be arrived at by extensive experiments aiming to find the load taken by the stiffened elements at the advanced post buckling stage of the unstiffened elements. A theoretical study, of course, is a possibility but it will be too involved.

For reasons mentioned elsewhere, the effective width calculated at any stress 'f' should be less for unstiffened elements of angle sections than of other sections. The interaction equation used for combined bending and axial force does not take into account the fact that the principle of super position does not hold good. This means that interaction will be dependant on the sequence of loading.

4.3 Comments on the Draft Revision of IS Handbook SP:6(5)-1970:

The draft revision of the IS Handbook for Structural Engineers Cold-Formed Light Gauge Steel Structures (SP:6(5)-1970)

is based entirely on Ref.(1) . Part I-Commentary aims at giving the theoretical background on which the specification is based . Although it succeeds in this to a large extent, it would be better if more theoretical aspects are given than actual derivations of equations. The Design Charts and Tables (Part II) is extensive and hence will be of great value in reducing the time required for a design.

In Part III (Design Examples) certain approximation are made viz. the rounded corners are considered to be right angled corners for ease of computation. But this approximation has not been suggested in the part of the Commentary where the moment of inertia of a rounded corner is derived. As the thicknesses of sections (elements) are of the order of 1-2.5 mm, it will be easier for the designer to consider the whole section as a line and arrive at the design. The value of the stress on the extreme fibres can be taken to be the stress acting on the centre line of extreme fibre. There should also be atleast a passing reference to the situation described in Chapter 3 herein.

4.4 Suggestions for Future Work:

It is worthwhile to investigate further the following topics:

(i) The effective width equation for unstiffened elements of angle sections.

(ii) The effect of stiffening offered by the unstiffened elements in their post buckling stage.

(iii) The effect of the sequence of loading on the interaction between moment and axial force.

APPENDIX

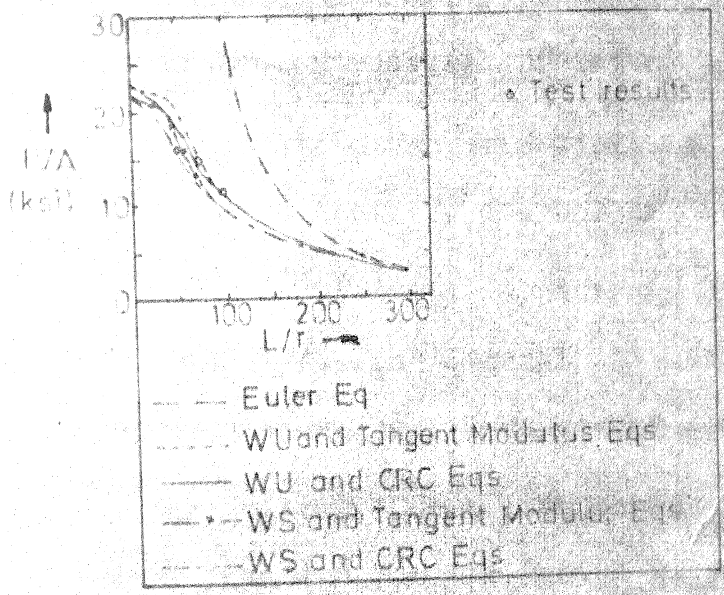

FIG. A-1

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